An *Euler path* in a graph G is a simple path (no repeated edges) containing every edge of G. An *Euler circuit* is an Euler path beginning and ending at the same vertex. We have two theorems about when these exist:

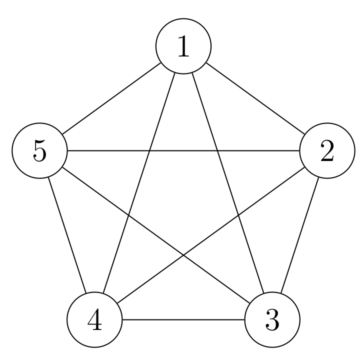
1. A connected graph G with at least 2 vertices has an Euler circuit iff each vertex has even degree.
2. A connected graph G hass an Euler path that is not an Euler circuit iff it has exactly two vertices of odd degree (and all other vertices have even degree).

Exercises

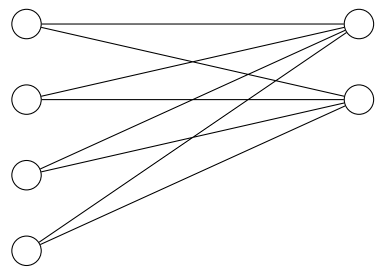
1. Why are these theorems false if G is not connected?

2. Show that having an Eulerian circuit (or path) is a graph invariant.

3. Which complete graphs K_n have Eulerian circuits? Find an Eulerian circuit in K_5:



4. Find an Eulerian circuit in the following graph:



5. If $G = (V, E_1)$ and $H = (V, E_2)$ have Eulerian circuits, which of the following also have Eulerian circuits? (Hint: use the theorems about the degree. What can the degrees of the vertices be in each of the following situations?)

   (a) $G_1 = (V, E_1 \cup E_2)$

   (b) $G_2 = (V, E_1 \cap E_2)$

   (c) $G_3 = (V, E_1 \oplus E_2)$, assuming G_3 is connected. (remember $A \oplus B = (A \backslash B) \cup (B \backslash A)$ is all the elements in A or B, but not in both.)