Generalized Permutations

Problems of this type tend to look something like: how many ways are there to permute n objects, where there are n_1 indistinguishable objects of type 1, n_2 objects of type 2, and so on. (The answer is $\frac{n!}{n_1!n_2!\cdots n_k!}$.)

1. How many permutations are there of the word 'COMBINATORICS'?

2. How many integers can be formed using all the digits $1, 3, 3, 3, 4, 5, 9$?

3. How many ways are there to line up 3 apples, 4 bananas, 5 oranges, and 6 kiwis?

Combinations with Repetition

There's a bit more variety with these types of problems. They all boil down to the question: how many ways are there to select k elements from an n element set if repetition is allowed? (The answer is $\binom{n+k-1}{k}$.)

1. Suppose you want to buy 10 pieces of fruit. If you can choose among mangos, dragon fruit, coconuts, and kumquats, how many ways are there to choose 10 pieces of fruit?

2. How many ways if you want to buy at least 1 of each fruit?

3. How many ways if you don't want more than 2 coconuts?

4. Restate problems $1-3$ using the format of finding the number of solutions of a linear equation. (i.e. something like: How many solutions does the equation $\sum_i x_i = n$ have where $a_i \leq x_i \leq b_i$?)

5. How many ways are there to place n indistinguishable objects into k distinguishable boxes?

More Combinatorial Proofs

1. Prove that $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$ using a combinatorial proof.

2. Prove $\sum_{k=0}^{n} \binom{k}{c} = \binom{n+1}{c+1}$ $(n, c \in \mathbb{N})$ using both a combinatorial and an inductive proof.

3. Prove that $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$ using a combinatorial proof.