More Induction

1. Assume you only know that $\frac{d}{dx}(x^0) = 0$ and also that the product rule is true. Is it possible to use induction to prove $\frac{d}{dx}(x^n) = nx^{n-1}$ for all integers $n \geq 0$?

2. Now assume you only know that $\frac{d}{dx}(x) = 1$ and also that the product rule is true. Now use induction to prove $\frac{d}{dx}(x^n) = nx^{n-1}$ for all integers $n \geq 1$.

3. Is it possible to extend your proof above to all real numbers?

4. Show that $1(1!) + 2(2!) + \cdots + n(n!) = (n+1)! - 1$ for all integers $n \geq 0$.

5. Show that $9 \mid 4^n + 15n - 1$ for all integers $n \geq 0$. (Tricky...)

Recursive Definitions

1. Recall that an arithmetic progression is of the form $a_n = a + nd$, where $a, d \in \mathbb{R}$ and $n \in \mathbb{N}$. Write this sequence using a recursive definition.

2. Recall that a geometric progression is of the form $a_n = ar^n$, where $a, r \in \mathbb{R}$ and $n \in \mathbb{N}$. Write this sequence using a recursive definition.

3. Give a recursive definition of the set $\mathbb{N}$.

4. Recall the Fibonacci sequence defined by $f_0 = 0, f_1 = 1$, and $f_n + f_{n+1} = f_{n+2}$ for all $n \in \mathbb{N}$. Prove (using induction) that $f_0 f_1 + f_1 f_2 + \cdots + f_{2n-1} f_{2n} = f_{2n}^2$ for $n \in \mathbb{Z}^+$.

5. Prove (using induction) that $f_{n-1} f_{n+1} - f_n^2 = (-1)^n$ for all $n \in \mathbb{Z}^+$.