Use implicit differentiation to find y' (or $\frac{dy}{dx}$) for the following:

1. $\sin(xy) = y$

   Start by taking the derivative:
   $$(\sin(xy))' =' y$$
   To deal with the left hand side, we need to apply the chain rule:
   $$\cos(xy) \cdot (xy)' = y'$$
   To finish the chain rule, we need to apply the product rule:
   $$\cos(xy) \cdot (x'y + xy') = y'$$
   $$\cos(xy) \cdot (1 * y + xy') = y'$$
   To finish, we need to solve for y' by isolating all the terms containing y' on one side:
   $$\cos(xy)y + \cos(xy)xy' = y'$$
   $$\cos(xy)y = y' - \cos(xy)xy'$$
   Finally, factor out y' from the right hand side, and divide by the remaining factor:
   $$\cos(xy)y = y'(1 - \cos(xy)x)$$
   $$y' = \frac{\cos(xy)y}{1 - \cos(xy)x}$$

2. $x^2 + xy + y^{15} = y$

   Start by taking the derivative:
   $$(x^2 + xy + y^{15})' = y'$$
   To deal with the left hand side, we apply the product and chain rules:
   $$2x + (xy)' + (y^{15})' = y'$$
   $$2x + (1)y + xy' + 15y^{14}y' = y'$$
   Isolate y', factor it out, and divide by the remaining factor:
   $$2x + y = y' - xy' - 15y^{14}y'$$
   $$2x + y = y'(1 - x - 15y^{14})$$
   $$y' = \frac{2x + y}{1 - x - 15y^{14}}$$

3. $\cos(x^2y) = x$

   Start by taking the derivative:
   $$(\cos(x^2y))' = x'$$
   To deal with the left hand side, we apply the chain rule:
   $$-\sin(x^2y) \cdot (x^2y)' = 1$$
   We apply the product rule on the left hand side to finish taking the derivative:
   $$-\sin(x^2y) \cdot (2xy + x^2y') = 1$$
   To isolate y', first divide by $-\sin(x^2y)$:
   $$2xy + x^2y' = \frac{-1}{\sin(x^2y)}$$
   Then subtract $2xy$ from both sides:
   $$x^2y' = \frac{-1}{\sin(x^2y)} - 2xy$$
   And finally, divide both sides by x^2:
   $$y' = \frac{-1}{x^2\sin(x^2y)} - \frac{2y}{x}$$

4. $e^{y^2} = y$

   Start with the derivative:
   $$(e^{y^2})' = y'$$
   Apply the chain rule to the left hand side:
   $$e^{y^2} \cdot (y^2)' = y'$$
   Apply the chain rule again to compute the derivative of y^2:
   $$e^{y^2}(2yy') = y'$$
   Subtract $e^{y^2}(2yy')$ from both sides:
   $$0 = y' - e^{y^2}(2yy')$$
   Factor out y':
   $$0 = y'(1 - e^{y^2}(2y))$$
   Divide both sides by $1 - e^{y^2}(2y)$:
   $$0 = y'$$

5. $\tan(y\cos(y)) = x$

   Start with the derivative:

   $$(\tan(y\cos(y)))' = x'$$

   Apply the chain rule to the left hand side:

   $$\sec(y\cos(y))^2 \cdot (y\cos(y))' = 1$$

   Apply the product rule to the left hand side:

   $$\sec(y\cos(y))^2 \cdot (y'\cos(y) - y\sin(y)y') = 1$$

   To begin isolating y', divide both sides by $\sec(y\cos(y))^2$:

   $$y'\cos(y) - y\sin(y)y' = \frac{1}{\sec(y\cos(y))^2}$$

   Factor out y' from the left hand side:

   $$y'(\cos(y) - y\sin(y)) = \frac{1}{\sec(y\cos(y))^2}$$

   and divide both sides by $\cos(y) - y\sin(y)$:

   $$y' = \frac{1}{\sec(y\cos(y))^2} * \frac{1}{\cos(y) - y\sin(y)}$$