

1. False. There are usually multiple ways to perform integration by parts.
2. True. Sometimes, our approximation may be equal to the function itself.
3. (a) First, we use u -sub to simplify the problem. Let $u = x^3$. Then $du = 3x^2 dx$, so $\frac{du}{3} = x^2 dx$. Now the problem is $\int \frac{ue^u}{3} du = \frac{1}{3} \int ue^u du$.
Now we need to solve this problem using integration by parts. Set $f(u) = u$ and $g'(u) = e^u du$. Then we get $f'(u) = du$ and $g(u) = e^u$.
Then $\frac{1}{3} \int ue^u du = \frac{1}{3}(ue^u - \int e^u du) = \frac{1}{3}(ue^u - e^u + C)$. Now we have to plug back in our substitution: $\frac{1}{3}(x^3 e^{x^3} - e^{x^3} + C)$.
- (b) The error E_T for trapezoid rule is bounded by $\left| \frac{C(b-a)^3}{12n^2} \right|$, where $C = \left| \max_{x \in [1,2]} f''(x) \right|$.
The second derivative of $\ln(x)$ is $-\frac{1}{x^2}$. The maximum of $\left| -\frac{1}{x^2} \right|$ occurs at $x = 1$, and the value is 1. $a = 1$ and $b = 2$, so $b - a = 1$.
We want $\frac{1 \cdot (1)^3}{12n^2} \leq \frac{1}{12 \cdot 100}$. Solving for n gives $n \geq 10$.