charles wang quiz 3 solutions

- 1. True. This is the definition of a removable discontinuity.
- 2. False, this limit evaluates to e.
- 3. (a) Plug in to the limit definition:

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{1}{(3+h)^2 + 1} - \frac{1}{3^2 + 1}}{h} = \lim_{h \to 0} \frac{\frac{1}{(3+h)^2 + 1} - \frac{1}{10}}{h}$$

Find a common denominator in the numerator.

$$= \lim_{h \to 0} \frac{\frac{10 - (3+h)^2 - 1}{10(3+h)^2 + 10}}{h} = \lim_{h \to 0} \frac{\frac{-6h - h^2}{10(3+h)^2 + 10}}{h} = \lim_{h \to 0} \frac{-6 - h}{10(3+h)^2 + 10} = \frac{-6}{100}$$

(b) Apply the chain rule with f(x) = g(h(i(x))) where  $g(x) = \ln(x), h(x) = \tan(x)$ , and  $i(x) = \frac{1}{x}$ .  $f'(x) = g'(h(i(x))) \cdot h'(i(x)) \cdot i'(x)$ .

$$g'(x) = \frac{1}{x}, h'(x) = \sec^2(x), i'(x) = \frac{-1}{x^2}$$
$$f'(x) = \frac{1}{\tan(\frac{1}{x})} \cdot \sec^2(\frac{1}{x}) \cdot \frac{-1}{x^2}$$