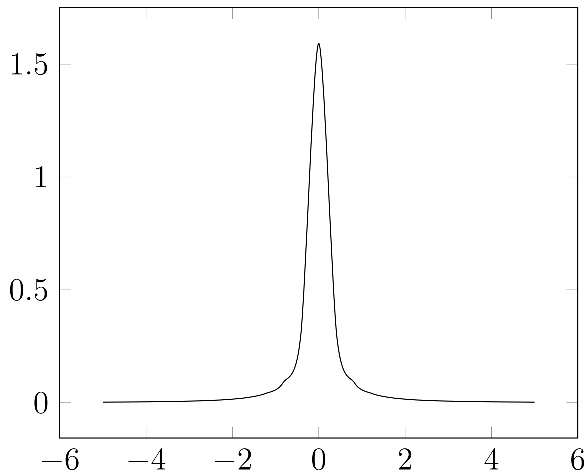
1. False. The converse statement, "A symmetric function has equal mean and median if the mean exists" is true, but this statement is not.

2. False. There can be no pdf because the integral $\int_{-\infty}^{\infty} f(x)dx$ will be either 0 or infinite.

3. (a) Here is the graph of $f(x)$.



(b) $f(x) \geq 0$ since x^2 is always positive, so we only need to check that $\int_{-\infty}^{\infty} f(x)dx = 1$.

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{5}{\pi} \frac{1}{1+25x^2} dx$$

Use the substitution $u = 5x$. Then $du = 5dx$, so the integral becomes:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1+u^2} du$$

and this is number 14 on the homework, so it is a pdf. If you don't remember number 14, do the following:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1+u^2} du = \frac{1}{\pi} \left(\lim_{t \to -\infty} \int_{t}^{0} \frac{1}{1+u^2} du + \lim_{t \to \infty} \int_{0}^{t} \frac{1}{1+u^2} du \right)$$

The antiderivative of $\frac{1}{1+u^2}$ is $\arctan(u)$, which has horizontal asymptotes at $y = \frac{\pi}{2}$ and $y = \frac{-\pi}{2}$, so the previous integral is:

$$\frac{1}{\pi} \left(\lim_{t \to -\infty} (\arctan(0) - \arctan(t)) + \lim_{t \to \infty} (\arctan(t) - \arctan(0)) \right)$$

$$= \frac{1}{\pi}(0 - \frac{-\pi}{2} + \frac{\pi}{2} - 0) = \frac{\pi}{\pi} = 1$$

(c) Since the function is symmetric about $x = 0$, the median is 0. The mean does not exist (see number 14 on the homework, it's almost the same computation).