1. False. $\int_{-\infty}^{\infty} f(x)dx$ may not be convergent, and then no nonzero constant factor can make it convergent. (If $c = 0$, then the integral will not be 1.)

2. False. pdf's can be discontinuous. (For example, maybe there is some strict upper limit on the values a random variable can take, and then the probability is zero otherwise.)

3. (a)
$$\int_1^\infty f(x)dx = \lim_{a\to\infty} \int_1^a \frac{1}{x^3}dx = \lim_{a\to\infty} -\frac{1}{2}\frac{1}{x^2}\Big|_1^a = \lim_{a\to\infty} -\frac{1}{2}(\frac{1}{a^2} - \frac{1}{1}) = \frac{1}{2}$$
Thus we should pick $c = 2$.

   (b)
$$F(x) = \int_1^x cf(t)dt = \int_1^x \frac{2}{t^3}dt = -\frac{1}{t^2}\Big|_1^x = -\frac{1}{x^2} - (-\frac{1}{1}) = 1 - \frac{1}{x^2}$$
Compute the median by $0.5 = F(x) = 1 - \frac{1}{x^2} \implies \frac{1}{x^2} = \frac{1}{2} \implies x = \sqrt{2}$