1 Antiderivatives

1.1 T/F

1. (True False) Every function has an antiderivative.
2. (True False) Every continuous function has an antiderivative.
3. (True False) Every differentiable function has an antiderivative.
4. (True False) Antiderivatives are usually harder than derivatives.
5. (True False) The antiderivative of the derivative of a function is equal to the function itself.
6. (True False) The derivative of an antiderivative of a function is equal to the function itself.

1.2 Exercises

Find the antiderivatives of the following functions (you may need to "undo" some derivative rules):

1. 0 (the constant 0 function)
2. $x^{-204.5}$
3. $\frac{1+x^2+x^3}{x^7}$
4. $\frac{1}{2\sqrt{x}}$
5. 10^x
6. $\sec(x)\tan(x)$
7. $e^{\sin(x)}\cos(x)$
8. $x\cos(x^2)$
9. $\sin(x)\cos(x)$
10. $\frac{1}{4+x^2}$
11. $\frac{\sin(x)^3+\sin(x)\cos(x)^2}{\cos(x)^2}$
12. $\csc(x)\cot(x)$

2 Riemann sums

2.1 T/F

1. (True False) Riemann sums always exist for any function.
2. (True False) Let R_n be the Riemann sum for a function $f(x)$ on the interval $[a, b]$ using n evenly spaced rectangles. The limit, $\lim_{n\to\infty} R_n$, if it exists, is the area under the curve of $f(x)$ from a to b. (If the limit exists, $f(x)$ is called *integrable* on $[a, b]$.)
3. (True False) Riemann sums using right endpoints generally overestimate the area of a function.
4. (True False) Riemann sums using left endpoints generally underestimate the area of a function.
5. (True False) Riemann sums are not useful because they are so difficult to compute.
6. (True False) Finite Riemann sums always exist for any function.

2.2 Exercises

1. Compute R_4 (same meaning as above) for $f(x) = x^2$ on $[0, 1]$ with left endpoints. Does this overestimate or underestimate the actual area? (For the last part, a picture is helpful.)
2. Compute R_4 (same meaning as above) for $f(x) = x^2$ on $[0, 1]$ with right endpoints. Does this overestimate or underestimate the actual area?
3. Think about when right Riemann sums will over/underestimate the area. Same with left Riemann sums.
4. Compute L_4 for $f(x) = e^{|x|}$ on $[-1, 1]$ with right endpoints. Do it with left endpoints.
5. Let $f(x)$ be the function defined by:

   $$f(x) = \begin{cases} 0 & x \in \mathbb{Q} \\ 1 & x \notin \mathbb{Q} \end{cases}$$

   Compute R_{1000} for $f(x)$ on $[0, 1]$ with right endpoints. Compute R_{1000} for $f(x)$ on $[0, \pi]$ with right endpoints.

   Do you think the Riemann sum $\lim_{n\to\infty} R_n$ exists? Why or why not?

3 Definite Integrals, FTC

3.1 T/F

1. (True False) Riemann sums are only useful for computing definite integrals because they cannot be used to compute antiderivatives.
2. (True False) Our integration techniques only apply to definite integrals, so we can't apply them to indefinite integrals.
3. (True False) The FTC connects Riemann sums with antiderivatives.
4. (True False) The interpretation of a definite integral as the area under the curve is not useful because we can almost never compute this area.
5. (True False) Initial conditions are necessary to solve definite integral problems.
6. (True False) We can ignore the "$+C$" for definite integrals because the constant disappears when solving the problem.

3.2 Exercises

1. Let $F(x) = \int_x^{e^x} e^{x^2} dx$. Compute $F'(x)$.
2. Compute $\int_0^1 \sin(x) dx$.
3. If it exists, compute $\int_0^1 \frac{1}{x} dx$. If it does not exist, state why.
4. Compute $\int_{-4}^4 \sqrt{16 - x^2} dx$.
5. A bacterial colony begins with 500 cells. The number of cells doubles every hour. Give a function $g(x)$ whose value at x is the number of cells at time x.

4 u-substitution, Integration by Parts

4.1 T/F

1. u-substitution undoes the chain rule of differentiation.
2. Integration by Parts undoes the product rule of differentiation.
3. Integration by parts always makes a problem simpler immediately.
4. The rule for u-substitution can only be derived by using Riemann sums.
5. Integration by parts will always solve problems in one step.
6. u-substitution cannot be used in definite integrals because we cannot predict how the limits of integration will change.
7. u-substitution may result in different antiderivatives.

4.2 Exercises

1. Find $\int \sin(x)\cos(x)dx$ using u-substitution with $u = \sin(x)$ and $u = \cos(x)$. Compare your answers, and explain why they look different.
2. Find $\int \sqrt[3]{1+\sqrt{x}}dx$.
3. Find $\int \frac{x^5}{x^2+4}dx$.
4. Find $\int_0^9 \sqrt{4-\sqrt{x}}dx$.
5. Find $\int x^3 e^x dx$.
6. Find $4x\cos(2-3x)dx$.
7. Find $\int e^x \sin(x)dx$.