

1. False. This limit evaluates to \sqrt{e} .
2. True. A function which is not continuous is automatically not differentiable.
3. (a) Use the definitions and plug in.

$$f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{(5+h) + (5+h)^2} - \sqrt{5+5^2}}{h}$$

Multiply the numerator and denominator by the conjugate of the numerator:

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\sqrt{(5+h) + (5+h)^2} - \sqrt{5+5^2}}{h} \frac{\sqrt{(5+h) + (5+h)^2} + \sqrt{5+5^2}}{\sqrt{(5+h) + (5+h)^2} + \sqrt{5+5^2}} \\ &= \lim_{h \rightarrow 0} \frac{(5+h) + (5+h)^2 - 5 - 5^2}{h(\sqrt{(5+h) + (5+h)^2} + \sqrt{5+5^2})} \\ &= \lim_{h \rightarrow 0} \frac{h + 2 * 5h + h^2}{h(\sqrt{(5+h) + (5+h)^2} + \sqrt{5+5^2})} \\ &= \lim_{h \rightarrow 0} \frac{1 + 2 * 5 + h}{\sqrt{(5+h) + (5+h)^2} + \sqrt{5+5^2}} \\ &= \frac{1 + 2 * 5}{2\sqrt{5+5^2}} = \frac{11}{2\sqrt{30}} \end{aligned}$$

- (b) Apply the chain rule with $f(x) = g(h(i(x)))$ where $g(x) = \sin(x)$, $h(x) = \cos(x)$, and $i(x) = \sqrt{x}$. $f'(x) = g'(h(i(x))) \cdot h'(i(x)) \cdot i'(x)$.

$$g'(x) = \cos(x), h'(x) = -\sin(x), i'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \cos(\cos(\sqrt{x})) \cdot (-\sin(\sqrt{x})) \cdot \frac{1}{2\sqrt{x}}$$