

1. False. The function could be negative in some places, and positive in others (for example $f(x) = \frac{\sin(x)}{x^2}$ on $(0, \infty)$) and then no single constant would work.

2. False. The function must also be nonnegative.

3. (a)

$$\int_1^\infty \frac{1}{x^4} dx = \lim_{a \rightarrow \infty} \int_1^a \frac{1}{x^4} dx = \lim_{a \rightarrow \infty} -\frac{1}{3} \frac{1}{x^3} \Big|_1^a = \lim_{a \rightarrow \infty} -\frac{1}{3} \left(\frac{1}{a} - \frac{1}{1} \right) = \frac{1}{3}$$

Therefore we should pick $c = 3$, so that $\int_1^\infty cf(x) = 1$.

(b)

$$F(x) = \int_1^x cf(t) dt = \int_1^x \frac{3}{t^4} dt = -\frac{1}{t^3} \Big|_1^x = -\frac{1}{x^3} - \left(-\frac{1}{1} \right) = 1 - \frac{1}{x^3}$$

To find the median, we want $0.5 = F(x) = 1 - \frac{1}{x^3} \implies \frac{1}{x^3} = \frac{1}{2} \implies x = \sqrt[3]{2}$.