name : _____

section : 105

GSI : Charles Wang

(2 pts) Circle True or False. (+1 for correct, 0 for blank, -1 for incorrect)

1. (True False) Using the comparison test with $f(x) = \frac{1}{x}$ can determine that an improper integral is convergent.

2. (True False) Differential equations usually have a unique solution, with or without an initial condition.

(10 pts) For the following, you must **justify** your answer to receive credit. (Showing your work counts as justification.)

3. (a) Does $\int_1^\infty e^{-x^2}\,dx$ converge or diverge? (Hint: $e^{x^2} > x^2$).

(b) The temperature, $T(x)$, of an object changes at a rate equal to the square of the difference between the object's temperature $T(x)$, and the surrounding temperature, A, which is a constant.

i. (2 pts) Write a differential equation $\frac{dT}{dx}$ modelling this situation.

ii. (3 pts) Solve this differential equation, if the initial temperature $T(0) = 96$ and $A = 75$.